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Seat No. _______________

HAL-003-1015003

B. Sc. (Sem. V) (CBCS) (W.E.F.-2016) Examination

June – 2023

Mathematics : Paper-7(A)

(Boolean Algebra & Complex Analysis-I)

Faculty Code : 003
Subject Code : 1015003

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions :

(1) All questions are compulsory.
(2) Figures to the right indicate full marks of the question.

1 (A) Answer the following questions : **4**

(1) Define : Least element in a partially ordered set.
(2) In a lattice $(L,\leq)$, for any $a,b\in L, a\oplus(a*b)=$ _______.
(3) Define : Lattice homomorphism.
(4) Define : Bounded lattice.

(B) Attempt any **one** : **2**

(1) Draw Hasse diagram of (S_6, D).
(2) Let $(L,\leq)$ be a lattice. Prove that for any $a,b\in L, a\leq b$ if and only if $a*b=a$.

(C) Attempt any **one** : **3**

(1) Let $(L,\leq)$ be a lattice. For any $a,b,c\in L$, prove that $a*(b*c)=(a*b)*c$.
(2) Define : Complete lattice. Prove that every finite lattice is complete.

(D) Attempt any **one** : **5**

(1) Prove that direct product of two lattices is a lattice.
(2) State and prove De Morgan's laws for a complemented distributive lattice.

2 (A) Answer the following questions : **4**

(1) Boolean algebra is a _______ and ________ lattice.

(2) Write atoms of boolean algebra $(S_{30},*,\oplus,',1,30)$.

(3) Define : Boolean homomorphism.

(4) How many minterms can be formed using n variables ?

(B) Attempt any **one** : **2**

(1) Let $(B,*,\oplus,',0,1)$ be a boolean algebra. Prove that a nonempty subset S of B is subboolean algebra of B if S is closed under $*$ and '.

(2) Reduce the boolean expression $\alpha(x,y)=(x*y)'\oplus(x\oplus y)$.

(C) Attempt any **one** : **3**

(1) Express the boolean expression $f(x_1,x_2,x_3)=x_1x_2$ as a product of sum canonical form.

(2) Find cube-array representation of the boolean expression $xy+xz'$.

(D) Attempt any **one** : **5**

(1) For any boolean algebra $(B,*,\oplus,',0,1)$, in usual notations prove that $A(x_1*x_2)=A(x_1)\cap A(x_2), \forall x_1,x_2\in B$.

(2) State and prove Stone's representation theorem.

3 (A) Answer the following questions. **4**

(1) What is real part of $f(z)=z^2$?

(2) Limit of function of complex variables need not be unique. (True/False)

(3) Determine the points at which the function $f(z)=\dfrac{1}{z^2+1}$ is discontinuous.

(4) Define : Analytic function.

(B) Attempt any **one** : **2**

(1) Using definition of limit, prove that

$$\lim_{z\to z_0}(az+b)=az_0+b.$$

(2) Prove that $f(z)=e^z$ is entire function.

(C) Attempt any **one** : **3**

(1) Prove that $\lim\limits_{z \to z_0} \frac{\bar{z}}{z}$ does not exist.

(2) Determine the analytic function $f(z) = u + iv$, where $u(x, y) = \sin x \cosh y$.

(D) Attempt any **one** : **5**

(1) Show that the function

$$f(z) = \begin{cases} |z|^2; z \neq 0; \\ 0; \quad z = 0 \end{cases}$$

is differentiable only at origin.

(2) State and prove necessary condition for a function $f(z)$ to be analytic.

4 (A) Answer the following questions : **4**

(1) Show that $f(z) = xy + iy$ is nowhere analytic.

(2) State Cauchy-Riemann equations in polar form.

(3) State Cauchy-Goursat theorem.

(4) If C represents the unit circle then $\int_C \frac{1}{2} dz =$ ______.

(B) Attempt any **one** : **2**

(1) Using Cauchy-Riemann equations in polar form, prove that the function $f(z) = \frac{1}{z} (z \neq 0)$ is analytic in its domain.

(2) If C is the arc of the circle |z| = 2 lying in the first quadrant, then find the upper bound of $\int_C \frac{1}{z^2 - 1} dz$.

(C) Attempt any **one** : **3**

(1) If $f(z) = u + iv$ is analytic function then prove that u and v satisfy Laplace equation.

(2) If $f(z) = u + iv$ is analytic function and u is constant then prove that $f(z)$ is constant.

(D) Attempt any **one** : **5**

(1) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function then prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} | f(z) |^2 = 4 | f'(z) |^2$.

(2) State and prove Cauchy's integral formula.

5 (A) Answer the following questions : **4**

(1) Which theorem is the converse of Cauchy-Goursat theorem ?

(2) sin z is bounded. [True/False]

(3) State Maximum modulus principle.

(4) What is the number of roots in $\mathbb{C}$ of the polynomial $z^5 + 1 = 0$?

(B) Attempt any **one** : **2**

(1) If C is unit circle, then evaluate $\int_C \frac{e^{2z}}{z^4} dz$.

(2) If $C : | z | = 1$, then evaluate $\int_C \frac{\sin z}{\left(z - \frac{\pi}{6} \right)^3} dz.$.

(C) Attempt any **one** : **3**

(1) State and prove Cauchy's inequality.

(2) State and prove Liouville's theorem.

(D) Attempt any **one** : **5**

(1) If $f(z)$ is analytic within and on simple closed contour C and z_0 is any point interior to C, then prove that $f'(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^2} dz$.

(2) State and prove Morera's theorem.
